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Identification and Linear Multivariable Control in an Absorption-Desorption Pilot Plant

A study of the identification of the dynamics of a pilot scale absorption-desorption system and the subsequent design of a multivariable control scheme are reported. A multivariable transfer function model was obtained from experimental input-output data in which the inputs (steam supply and liquid circulation rate) were perturbed simultaneously using uncorrelated pseudo random sequences. A two-step, least-squares estimator was employed for the identification of the model.

A multivariable controller was then designed on the basis of the direct Nyquist array method. The implementation of the controller design indicated the utility of the design procedure as compared to independent single input, single output control loops; there is, however, some scope for additional tuning.

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SCOPE

In recent years, several applications of modern control theory have been proposed for use in the chemical industry. However, despite the obvious theoretical advantages of these advanced strategies, their implementation in an on-line situation generally requires excessive computational effort. An alternative approach is the extension of well-proven frequency domain design methods to multivariable cases; this offers the advantages of easy implementation and the possibility of on-line adjustment or tuning of the controller. The design proceeds in two stages: a simple model of the system is first obtained, and a controller is then designed based on that model. In the linear case, it is possible to deduce from experimental input/output measurements a process model which represents the best linear fit of the nonlinear chemical process. It has been shown that the use of pseudo random binary sequences (PRBS) as input disturbances offers significant advantages over other forms of input signals such as square wave or sinusoidal excitation.

The multivariable controller can then be designed using one of several extensions of the Nyquist criterion to multivariable systems (Rosenbrock, 1974). Although

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these frequency domain techniques are theoretically less attractive than optimal controllers resulting from state space analysis, from a practical point of view their simplicity, high stability and low noise sensitivity may more than compensate for this. Only implementation on a realistic sized plant can decide the utility of the design.

CONCLUSIONS AND SIGNIFICANCE

Of all the applications of modern control theory, linear multivariable control is probably the simplest technique which can be used in an industrial environment. Frequency domain design of such controllers requires simple transfer function models of the process. In principle, these can be deduced (identified) from input/output measurements which do not disturb the plant's normal operating conditions excessively and do not require a separate detailed modeling exercise.

It is shown in this work that controller designs based on simple models can be successfully implemented on a large pilot scale absorption/desorption plant. The identification of transfer function models has heretofore been applied primarily to single input, single output systems, although simultaneous identification of multivariable models is more suitable for ultimate use in the design of multivariable controllers. The simultaneous excitation of all inputs for the multivariable identification experiment required plant running times of up to 8 hrs. Nevertheless, it is shown that despite the long experimental times required, identification algorithms can successfully be used on large plants with large time constants without disturbing plant performance.

The identification process led to models of second and fourth order (with time delay) for the dynamics of the system. The model was then used together with the direct Nyquist array technique to design a realizable controller which effectively decoupled the outputs while retaining desirable stability properties. Without doing any tuning of the controller, a significant improvement in the overall plant performance could be achieved when compared to the use of quasi independent controllers. Further small improvements could be achieved by alteration of the control constants from those predicted by the design method. This results primarily from the fact that a linear model has been used for a nonlinear process, and, hence, there is inevitably a deviation between the model and experiment.

In summary, it has been shown that multivariable controllers based on frequency domain design considerations can, in some situations, lead to significant improvement in control of large plants and require only a moderate investment of time and effort.

The availability of faster and more powerful computers has opened new possibilities for control system design and implementation on complex chemical plants. The use of state space techniques for control of laboratory scale equipment has been reported (Fisher and Seborg, 1976; Jutan et al., 1977; Kershenbaum and Fortescue, 1978), but from an engineering point of view, there is a good deal to be said for design procedures which are based on extensions of classical frequency response methods. Specifically, because they do not require a complete model of the plant, these methods generally involve an order of magnitude less effort in the modeling stages than controllers based on state space techniques. On the other hand, they will not lead to optimal controllers and are generally restricted to linear analysis.

In recent years, there have been a number of proposals for design methods for multivariable systems (Rosenbrock, 1969, 1971, 1972; MacFarlane, 1972; Mayne, 1973). The existing literature and frequency domain design methods are extensively reviewed by Rosenbrock (1974). Examples of the applications presented by Rosenbrock include processes with some nonlinear features such as furnaces or compressors. A large number of other developments and applications have been reported, including work by Horowitz and Shaked (1975), Mee (1976), Graham and Rang (1977), MacFarlane and Kouvaritakis (1977), Biltman and Ward (1978), Moore and Sagers (1978), Seraji (1979), Tyreus (1979), and at recent IFAC Symposia (1974, 1977).

In all cases, the implementation is an application of a linear multivariable controller which is based on a linearized process model. However, there are few examples in the literature of the application and evaluation of such design methods on industrial process plants. The work reported here is primarily concerned with such an application, in this case to a large pilot plant sized absorption-desorption system.

The first stage in any advanced control system design is the development of a process model. In the present case, where the requirement is for a linear model of a rather complex nonlinear multivariable chemical process, a number of possibilities exist ranging from methods based on deterministic physical models to those based on the identification of black box models. The application of identification methods has recently been reviewed by Gustavvson (1975), and a number of case studies as well as the theory are presented by Eykhoff (1973). One convenient method of building linear time invariant transfer function models on the basis of input-output data is by the use of pseudo random binary signal (PRBS) excitation of the system inputs. The use of these methods leads in theory to models representing the best linear fit of the plant operation. From a practical point of view, such methods have the advantage that when used in conjunction with a digital computer, the input disturbances can simply be implemented through shift registers (Briggs and Godfrey, 1966) with a minimum of effort. Because of the small amplitude of the excitation signals, these identification experiments can usually be carried out on operating plants with little or no disturbance of plant performance.

THEORY

For systems in which the inputs and outputs are sampled at discrete times rather than continuously, it is convenient to represent the dynamics in discrete form; in a linear, single input-single output (SISO) system, this becomes

$$y(k+d) + a_1 y(k+d-1) + \dots a_m y(k+d-m) = b_o u(k) + \dots + b_n u(k-n) + \xi(k+d) \quad (1)$$

These dynamics are usually represented in terms of the z transform to yield

$$y(k) = z^{-d}\left[\frac{B(z)}{A(z)} u(k)\right] + \xi(k)$$

where the backwards shift operator is defined as

$$z^{-1}\{x(k)\} \equiv x(k-1)$$

and A and B are polynomials in z^{-1}.

Similarly, application of the z transformation to a multivariable form of Equation (1) leads to

$$y_1(k) = z^{-d_1}\left[\frac{B_{11}}{A_{11}} u_1(k) + \frac{B_{12}}{A_{12}} u_2(k) + \dots + \frac{B_{1n}}{A_{1n}} u_n(k)\right] + \xi_1(k)$$

$$\vdots$$

$$y_p(k) = z^{-d_p}\left[\frac{B_{p1}}{A_{p1}} u_1(k) + \frac{B_{p2}}{A_{p2}} u_2(k) + \dots + \frac{B_{pn}}{A_{pn}} u_n(k)\right] + \xi_p(k) \quad (2)$$

where $y_1 \dots y_p$ are the model outputs, $u_1 \dots u_n$ are the model inputs, $\xi_1 \dots \xi_p$ are the model errors, $d_1 \dots d_p$ are the time delays and A_{ij} and B_{ij} are polynomials in Z^{-1}.

Unbiased estimates of the unknown polynomials A_{ij} and B_{ij} can be obtained from experimental input/output data only when the errors are uncorrelated white noise sequences. Clearly, in a simplified model of a real process plant, this will generally not be the case. To overcome these restrictions, we use the generalized least-squares technique, an iterative method which fits an autoregression model to the actual noise sequences. Thus, we assume that the error in the first output ξ_1 is dynamically related to a white noise sequence, $\xi_1 = D_{11}(z^{-1})\, e_1$, and errors in the other outputs are linear combinations of white noise sequences and are described by the set of difference equations

$$\xi_1 = D_{11} e_1$$

$$\xi_2 = D_{21} e_1 + D_{22} e_2$$

$$\vdots$$

$$\xi_p = D_{p1} e_1 + D_{p2} e_2 + \dots + D_{pp} e_p \quad (3)$$

where $e_1 \dots e_p$ are white noise sequences, and D_{ij} are unknown polynomials in z^{-1}.

This assumption leads to a noise transfer function matrix of lower triangular form. Hence, it allows for some noise interaction which will generally be present in real processes, but the equations are still simple enough to allow the computations to be done on a small process computer.

The generalized least-squares algorithm used in this work to estimate the unknown parameters is a two-step procedure, following the work of Abaza (1976). The first step filters the noise by estimating preliminary parameters γ_{ij} in a dynamic model:

$$y_i = z^{-d_i} \sum_{j=1}^{n} \gamma_{ij} u_j + \xi_i$$

where

$$\gamma_{ij} = \frac{B_{ij}}{A_{ij}}$$

The residuals ξ_i are then fitted by the autoregression model (3) by estimation of the parameters D_{ij}. The procedure can be carried out for $i=1, \dots n$ in a direct manner because of the lower triangular form of (3). The input/output measurements are then filtered using the autoregression model in an attempt to achieve uncorrelated white noise residuals via the transformation

$$y_i' = F_i(z^{-1})\, y_i$$

$$u_i' = F_i(z^{-1})\, u_i \quad (4)$$

where

$$F_i(z^{-1}) = 1/D_{ii}(z^{-1})$$

However, the inherent correlation of noise in the original data and the truncation of the polynomial F in Equation (4) both require that the procedure be iterated until the residuals in the model (2) are, in fact, uncorrelated white noise. Upon convergence of this first step, noise free output sequences $\hat{y}_i$ are calculated on the basis of the estimated parameters γ_{ij} and the inputs u_i.

The second step of the procedure involves the use of an ordinary least-squares algorithm with the filtered data to yield unbiased estimates of the parameters in the polynomials $A_{ij}(z^{-1})$ and $B_{ij}(z^{-1})$ in (2). Finally, the entire process is repeated with increased orders of the polynomials A and B until a statistical F-test reveals that no further increase in model order can be justified (Eykhoff, 1974).

The algorithm as presented above produces a discrete model in the z domain. In order to apply the direct Nyquist array method for controller design, two approaches have been used.

In one, the z transform model is related to the s domain directly via the identity $z \equiv e^{T_s s}$, where T_s is the sample time; this, however, requires a suitable expansion of the exponentials to yield a transfer function $G(s)$ in polynomial form.

Alternatively, a bilinear transformation can be applied which converts the discrete-time z transform model to a continuous representation in the w plane by taking

$$z = \frac{1+w}{1-w} \quad (5)$$

where w is a complex variable for a continuous transfer function model (see Luyben, 1973).

The advantage of the transformation (5) is that it maps the stability region in the z space (the interior of the unit circle) onto the open left-half plane in the w space. The close similarity of the stability criteria between the w space and the familiar s domain of the Laplace transformation allows the application of frequency design methods such as the Nyquist array, which are based on stability considerations for the system.

Upon transformation to the w domain, the process dynamics are now described in terms of a transfer function model which, for a two input–two output system takes the form

$$\bar{y}(w) = G(w)\, \bar{u}(w)$$

where

$$G(w) = \begin{bmatrix} g_{11}(w) & g_{12}(w) \\ g_{21}(w) & g_{22}(w) \end{bmatrix}$$

and in which for stability considerations w can be taken as equivalent to s. This latter approach utilizing analysis in the w domain has been used in this work.

Once a system model has been identified in the s domain (or equivalent), various extensions of the standard Nyquist stability criteria can be used to predict the stability of multivariable controllers. Control system design can

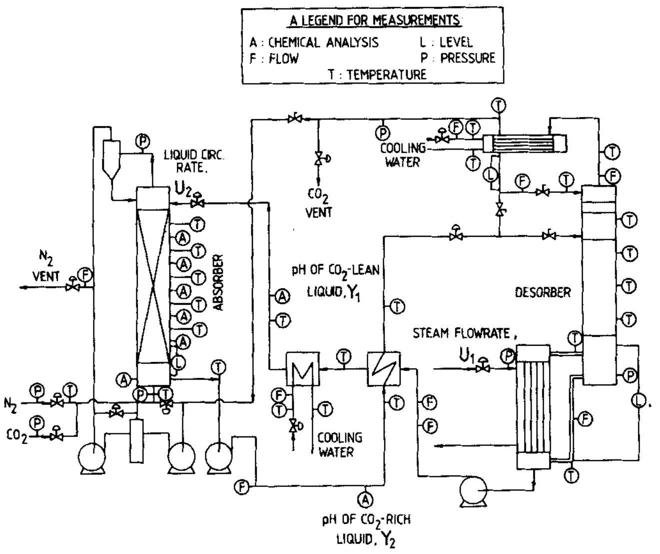

Figure 1. Schematic diagram of absorption-desorption system.

then be based upon gain and phase margin criteria as in the single input–single output case. These have been carried out using a computer aided control system design package developed at Imperial College (Denham, 1978).

The theoretical background to the control system design is outlined in detail by Rosenbrock (1974). The development is normally done in the s domain, but since the design criteria are based on stability considerations, they can be expressed in an identical manner in the w domain, with w replacing s.

Multivariable control system design based on the direct Nyquist array method requires a dominant system matrix $Q(s) = G(s)\ K(s)$, where $K(s)$ is the controller matrix which is to be designed and where all transfer function matrices are now expressed in the s domain. The closed loop transfer function matrix H is then given by

$$H(s) = [I + Q(s)]^{-1}\,Q(s) \tag{6}$$

In the above discussion, a dominant system matrix Q is defined as either

$$|q_{ii}| > \sum_{\substack{j=1\\ j\neq i}}^{m} |q_{ij}| \text{ or } |q_{ii}| > \sum_{\substack{j=1\\ j\neq i}}^{m} |q_{ji}|$$

The stability of the resulting control system can be related to the so-called Gershgorin circles associated with the matrix Q as discussed by Rosenbrock (1974). Gershgorin circles are circles with a radius

$$r_i(s) = \sum_{\substack{j=1\\ j\neq i}}^{m} |q_{ij}(s)|$$

for an arbitrarily selected frequency s on the Nyquist contour for q_{ii}. For a range of s, they sweep out a band consisting of a finite set of circles. If the band of circles excludes the point -1 on the negative real axis (and if the system is sufficiently diagonally dominant for stability to be inferred from the diagonal elements alone), then the resulting closed loop system is stable.

Practical experience has shown that a first approximation to the control matrix K to achieve dominance is a constant precompensator $K_1 = G^{-1}$, where G is evaluated at $s=0$ (or $w=0$). Additional elements K_2, K_3 may be required in order to achieve dominance and stability, where $K = K_1K_2K_3$. Here, K_2 and K_3 can be used to permute rows or columns, introduce integral action, etc. The use of these elements varies from one system to another, and a specific application is described later.

EXPERIMENTAL

The identification and control algorithms outlined above were implemented on a large pilot scale absorption-desorption system. The plant continously separates a nominal mixture of 5% carbon dioxide in nitrogen in a packed absorption tower (0.3 m diameter × 10 m height, packed with 25 mm ceramic pall rings). The absorbing liquid is normally a 15% solution of monoethanolamine in water. The solution is continuously regenerated in a desorption column equipped with a steam heated reboiler. The desorber has 20 sieve trays (0.25 m diameter, 25 mm weir height, 16 × 12.65 mm diameter holes per tray, 0.3 m tray spacing) with downcomers. The liquid circulates through the recirculation reboiler by free convection. Figure 1 gives a comprehensive picture of the available instrumentation. In addition, liquid samples can be taken from sample points along the absorber and the desorber and analyzed off line using a gas chromatograph.

The measurements include temperatures (indicated by T) obtained with platinum resistance thermometers, pressures (P) and liquid levels (L), obtained with differential pressure cells, flow rates (F) obtained by orifice plates and turbine meters and composition analyses (A) obtained by pH meters for the liquid phase and infrared analyzers for the gas phase.

The entire plant is controlled and supervised by a Honeywell DDP-516 computer with a 16-bit word length and 32K core memory (0.96 μs cycle time). The memory is assisted by a one million word backup disk. Available peripherals include teletypes, V.D.U.'s, high speed paper tape reader and punch, and digital plotter. Data acquisition and conventional SISO control are executed on line by incremental direct digital control (that is, velocity algorithms). The simplest control design uses up to 12 SISO control loops, each variable associated with a single valve. Process variables are connected to the computer using a real time interface. Output signals from the computer are transmitted to current/pressure converters and then to the pneumatic control valves. The computer system allows user written programs for data manipulation and processing to run simultaneously with the control executive in a foreground-background mode. These programs may be written in a special real time extension of Fortran IV. Although only 8 K of core space is available for special user programs, the present system is able to make use of disk resident programs which are core resident only during the time of their execution. This effectively increases the size of the system severalfold.

For the identification of appropriate system models, a two level PRBS generator was programmed which manipulates the two inputs: steam supply and the liquid circulation rate. Table 1 presents the operating parameters for simultaneous excitation using sixth-order PRBS signals.

In the experiments described, an attempt was made to control simultaneously the composition of the liquids (that is, percent carbon dioxide) leaving both columns; the available controls are

Table 1. Operating Parameters For Simultaneous PRBS Experiments

Sampling time = 30 s
Set point change of steam supply = 20% around 15 g/s
Set point change of liquid circulation = 10% around 300 g/s
Unit shift time (clock interval) for disturbance in steam flow = 2.5 min
Unit shift time (clock interval) for disturbance in liquid flow = 4 min
N_2 circulation in the absorber = 65 g/s
CO_2 concentration in the gas phase entering the absorber = 3%
Absorber pressure = 2.5 bar
Desorber pressure = 1 bar (atmospheric pressure)
Cooling water flow in condenser = 150 g/s
Cooling water flow in the heat exchanger = 150 g/s
Liquid level in the absorber = 0.4 m
Amine concentration = 0.049 (mole/mole)
Supply pressure of N_2 = 3-4 bar
Supply pressure of steam = 4-6 bar

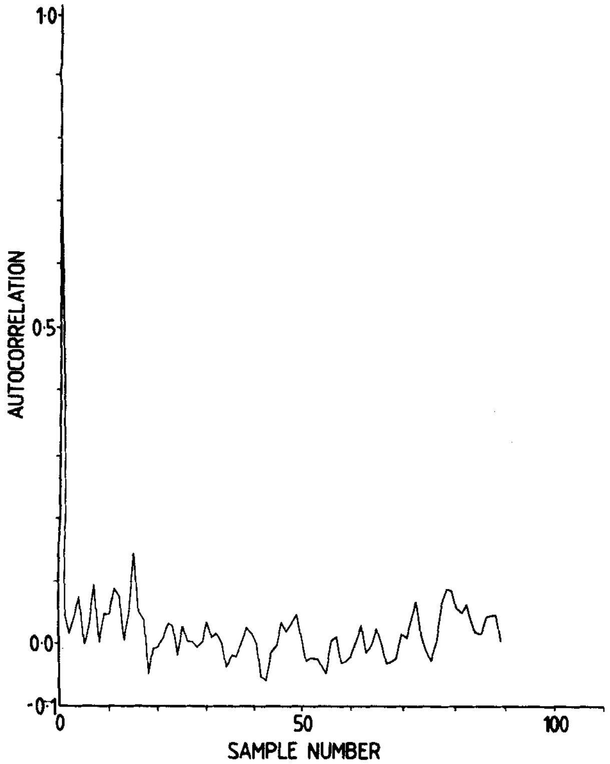

Figure 2. Normalized autocorrelation function for the error sequence ξ_1 (concentration of liquid leaving desorber).

the liquid circulation rate and the steam supply to the reboiler. Disturbances include variation in feed composition of the gas, pressure fluctuations and process instabilities caused by variable levels in the recirculation boiler.

In an absorption/desorption system, the two controls act in very different ways. Short-term disturbances can be overcome rapidly and effectively by varying the circulation rate of the liquid. Thus, small variations in feed gas composition can easily be dealt with by varying the liquid rate. However, ultimately, control of the system can only be achieved by varying the power input (that is, steam) to the regenerator.

This latter control is extremely slow because of the large hold-up and thermal capacity in the system. Clearly, a combination of the two controls will lead to the best overall system performance.

RESULTS AND DISCUSSION

Identification

The two-step algorithm described above was used to identify a multivariable linear z transform model from the response of the liquid concentrations to simultaneous PRBS disturbances of the main inputs, steam flow rate and liquid circulation rate. Because of the large time constants associated with the system, the time required for experimental runs extended up to 8 hrs.; special precautions were required to prevent excessive system drift caused by changes in ambient conditions. For this reason, direct perturbation of control valve positions was unsuitable and was replaced by a perturbation of set points in tight control loops around the input variables in question. All plant measurements were sufficiently stable not to require re-calibration and zeroing during a single run.

Since the system time constants and time delays are functions of flow rates (especially the liquid circulation rates), the amplitude of these perturbations was kept as low as the system sensitivity would permit. Thus, the steam flow rate was perturbed by 20% about a steady state value of 15 g/s and the liquid circulation rate by 10% about a value of 300 g/s. Smaller amplitude disturbances would be desirable in an industrial environment but would, of course, result in a lower sensitivity.

The model formulation as presented above allows for some degree of noise interaction. This can be tested by auto and cross correlations of the error sequences. Figure 2 shows the normalized autocorrelation of the error sequence ξ_1 (the error in the concentration leaving the desorber) calculated as

$$\Phi(L) = \frac{1}{\Phi(0)} \frac{1}{N-L} \sum_{k=1}^{N-L} \xi_1(k)\, \xi_1(k+L)$$

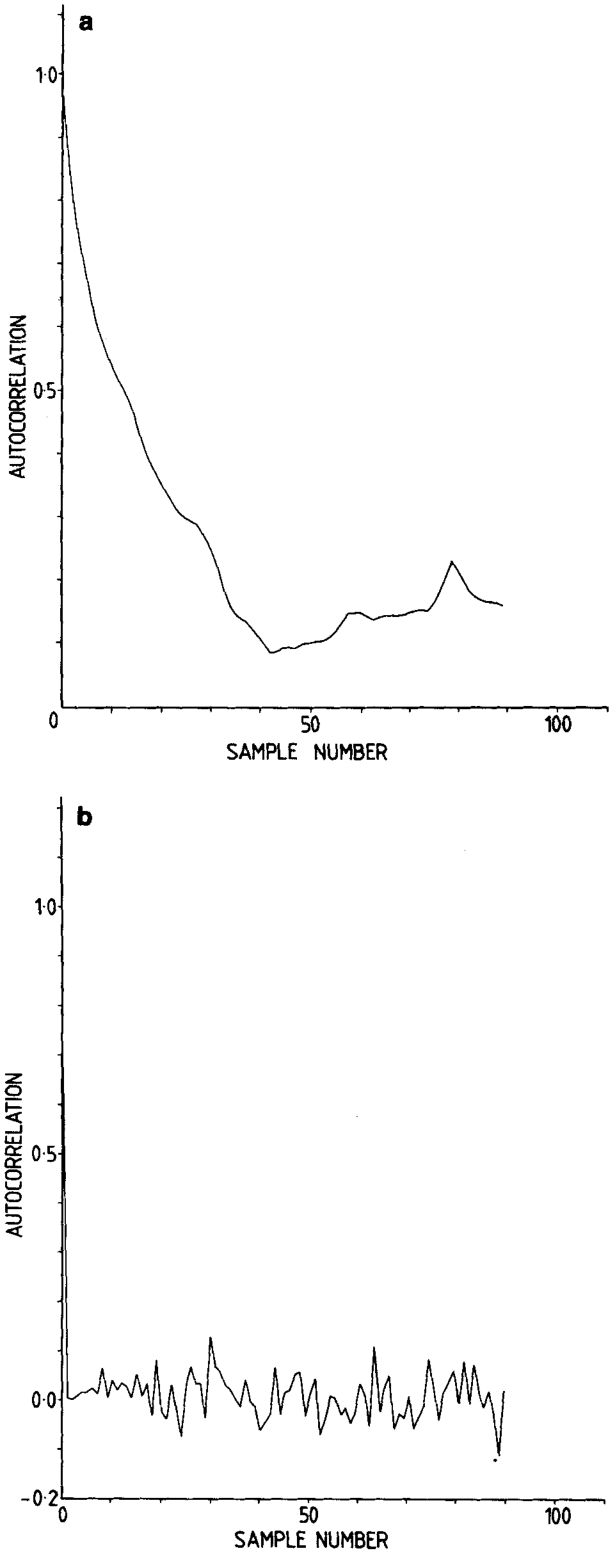

Figure 3. Normalized autocorrelation function for the error sequence ξ_2 (concentration of liquid leaving absorber: (*a*) before filtering (*b*) after filtering.

TABLE 2. NUMERICAL VALUE OF A MULTIVARIABLE z TRANSFORM MODEL, CORRESPONDING TO EQUATION (1)

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with u_1 = steam flow
u_2 = liquid circulation
y_1 = reboiler concentration
y_2 = absorber concentration
sample interval, T_s = 30 s

$g_{11}: d = 2$	$g_{12}: d = 2$
$b_0 = -0.5553 \times 10^{-3}$	$b_0 = 0.8032 \times 10^{-2}$
$b_1 = -0.4896 \times 10^{-3}$	$b_1 = 0.1302 \times 10^{-1}$
$b_2 = 0.4028 \times 10^{-4}$	$b_2 = 0.3278 \times 10^{-1}$
$a_1 = -0.6654$	$a_1 = -0.9917$
$a_2 = -0.1989$	$a_2 = 0.1212$

$g_{21}: d = 15$	$g_{22}: d = 5$
$b_0 = -0.533 \times 10^{-3}$	$b_0 = -0.1510 \times 10^{-1}$
$b_1 = -0.4328 \times 10^{-3}$	$b_1 = -0.3793 \times 10^{-1}$
$b_2 = 0.3792 \times 10^{-3}$	$b_2 = 0.1341 \times 10^{-2}$
$a_1 = -0.6899$	$b_3 = 0.1433 \times 10^{-1}$
$a_2 = -0.2199$	$a_1 = -1.3256$
	$a_2 = 0.2887$
	$a_3 = 0.1312$
	$a_4 = 0.0469$

Application of a statistical significance test shows that within the 95% confidence level, the residuals are uncorrelated, and the use of the autoregression model is not necessary.

Figure 3a, on the other hand, shows that the autocorrelation of the error sequence ξ_2, that for the composition of liquid leaving the absorber, is far from ideal. Hence, these input/output data have been filtered using an autoregression model of order 20. The filtered data then have an autocorrelation as illustrated in Figure 3b.

The change in the variance of the residuals ξ_2 indicates its convergence to a white noise sequence upon repeated application of the least-squares algorithm. The variance resulting from an initial application of linear least squares to the original data is 0.140×10^{-3}. After one application of the autoregression and subsequent filtering of the input/output data, the variance has dropped to 0.727×10^{-5} and after a second iteration has converged to 0.724×10^{-5}. Cross correlation of the sequences showed that ξ_1 and ξ_2 have no significant interaction (Albrecht, 1977).

Once the PRBS input/output data have been filtered, a linear identification algorithm is used to estimate parameters in the z transform polynomials A and B. The algorithm, however, requires the specification of model order before calculation can proceed.

In this work, the polynomial order for both A and B was increased until a statistical F test revealed that no further increase in model order was justified. The numerical values for the parameters of the identified model corresponding to Equation (1) are given in Table 2.

A comparison of the predictions of the identified model with the experimental response to simultaneous PRBS excitation of the inputs is illustrated in Figures 4a and b. It is clear that despite the high noise level in the system, the dynamics appear to be well represented by the simple model. (The high frequency noise in Figure 4a is caused primarily by instabilities associated with boiling in the reboiler.) Note, however, that the steady state gains in this nonlinear system are poorly represented by the linear model. This is not surprising, since the PRBS excitation is not designed to obtain very low frequency (or steady state) information. Furthermore, accurate knowledge of the gain is not crucial for control purposes.

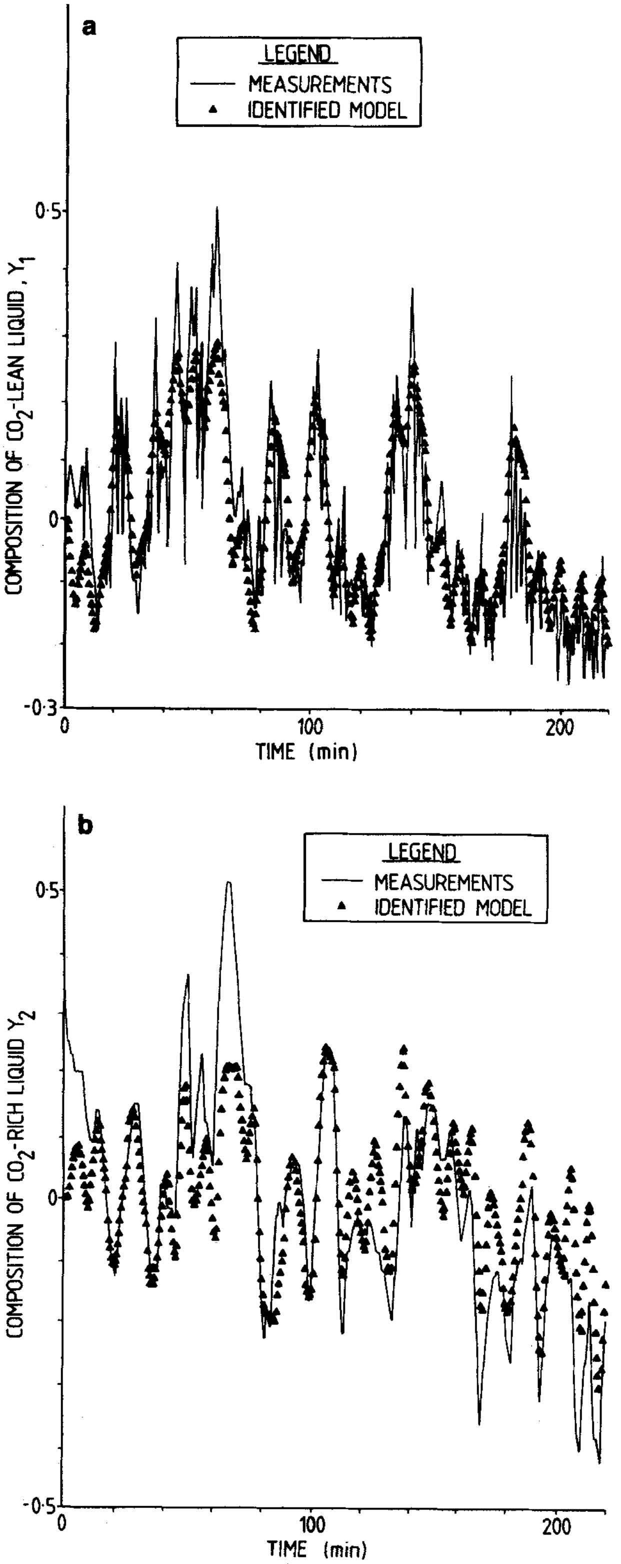

Figure 4. Simulated and experimental response to simultaneous PRBS excitation of both controls: (*a*) desorption column (*b*) absorption column.

Control system design and implementation

Only the implementation of a control system based upon the identified model can demonstrate the utility of the identification technique for control purposes. In order to apply familiar frequency design methods, the stability region of the time discrete z transform model was mapped into the open left-half plane of the w space.

Table 3 gives numerical values for the continuous w domain representation of the identified z transform mod-

Table 3. Representation Of A Discrete Time Model In A Continuous w Space

$$g_{11}(w) = \left(\frac{1-w}{1+w}\right)^2 \times \left[\frac{-0.10046 - 0.1192w - 0.002511w^2}{0.13559 + 2.19898w + 1.46644w^2}\right]$$

$$g_{12}(w) = \left(\frac{1-w}{1+w}\right)^2 \times \left[\frac{0.05384 - 0.0495w + 0.0278w^2}{0.13001 + 1.8787w + 2.11285w^2}\right]$$

$$g_{21}(w) = \left(\frac{1-w}{1+w}\right)^{15} \times \left[\frac{-0.093 - 0.11663w - 0.00705w^2}{0.095 + 2.212w + 1.4646w^2}\right]$$

$$g_{22}(w) = \left(\frac{1-w}{1+w}\right)^5$$

$$\times \left[\frac{-0.04005 - 0.165w - 0.0879w^2 + 0.04412w^3 + 0.007165w^4}{0.1422 + 0.8928w + 5.7937w^2 + 5.7324w^3 + 2.527w^4}\right]$$

The models in the w space are calculated after normalizing the input/output measurements to obtain dimensionless parameters. Measurements are normalized to a value between 0 and 1:

Concentrations:	range:	0 – 0.5	mole CO_2/mole amine
Steam flow:	range:	0 – 50	g/s
Liquid flow rate:	range:	0 – 0.5	kg/s

el. Note that the definition of inputs and outputs leads to negative gains for the diagonal elements of the transfer function matrix. The multivariable feedback control scheme is presented in Figure 5. The controller $K_1(s)$ is normally designed using, as a first approximation, $K_1 = G^{-1}(s = 0)$, or, in this case, evaluating G at $w=0$, to yield

$$K_1 = \begin{bmatrix} -0.4571 & -0.73108 \\ 1.5968 & -1.22 \end{bmatrix}$$

The Nyquist array for $Q(w) = K_1 G(w)$ as shown in Figure 6 indicates a strongly dominant system for the first row but not for the second row. For illustrative purposes, an asterisk marks the frequency 0.22 rad/s, where the entry for q_{22} comes closest to the critical point. The behavior of the second row can be attributed, at least partially, to the extreme difference in time delays in the system model, a delay of fifteen sample intervals for g_{21} and 5 for g_{22}. In addition to K_1, further compensation is necessary in order to obtain a fully dominant system.

As in the single variable case, frequency design methods offer no rigidly defined guidelines for obtaining the best controller; rather, there is considerable scope for choosing suitable control parameters within a loose framework. In the situation illustrated by Figure 6, increased diagonal dominance and an improvement in the control system can clearly be achieved by adding the second column to the first; that is,

$$K_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

In addition, implementation of any regulator in the form of a velocity algorithm (Smith, 1972) requires some integral control action to avoid drift and incorporate the set point explicitly. There is considerable freedom in the choice of the integral parameters as long as these are compatible with the Nyquist stability criteria. These are again most easily evaluated using the Gershgorin circles.

It is easy to show that the transfer function for a PI controller in the w domain is of the form $K_3(w) = a' + b'/w$, which is analogous to that of a continuous controller $K_3(s) = a + b/s$. If we express the final

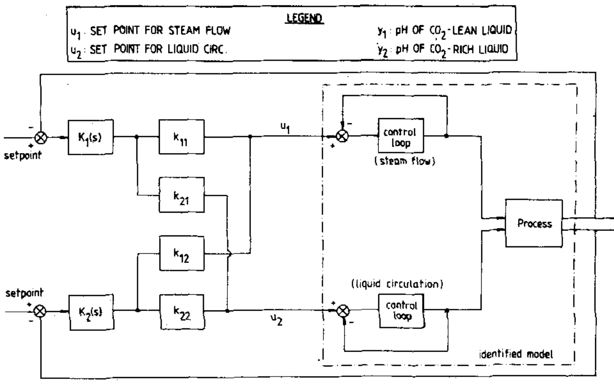

Figure 5. Scheme for multivariable control of absorption/desorption plant.

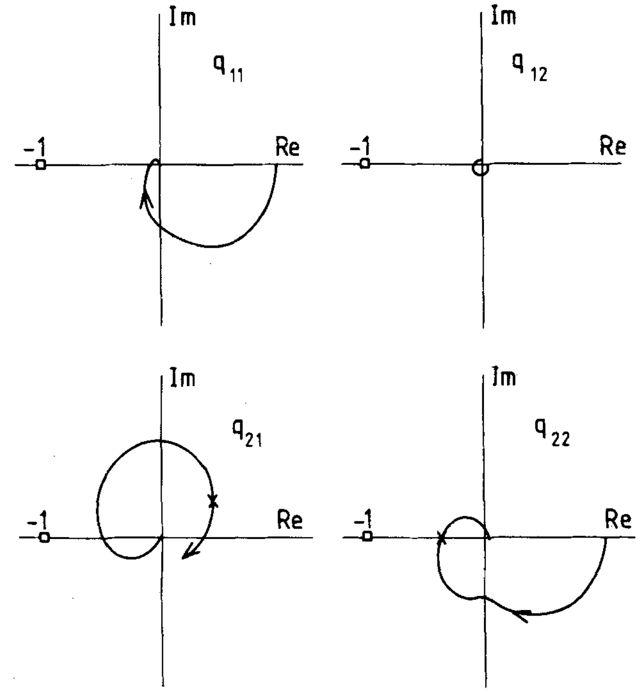

Figure 6. Nyquist array using constant precompensator, K_1.

result in the familiar s domain, with integral action of the form

$$K_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 + 0.01/s & 0 \\ 0 & 1 + 0.01/s \end{bmatrix},$$

the overall controller is given by $K = K_1 K_2 K_3$; that is

$$K = \begin{bmatrix} -0.594 & -0.7311 \\ 0.19 & -1.22 \end{bmatrix} \times \begin{bmatrix} 1 + 0.01/s & 0 \\ 0 & 1 + 0.01/s \end{bmatrix}$$

Here, a small amount of integral action has been introduced; the lower gain in the first diagonal element of K_3 has been introduced because of the high frequency noise in that loop (see Figure 4a). The resulting system matrix Q is illustrated in terms of its Nyquist array in Figure 7. The superimposed Gershgorin circles exclude the critical point -1, and indicate stability for all frequencies.

The multivariable linear control algorithm represented by K is equivalent to four PI controllers acting on the 2×2 system as illustrated in Figure 5. A program was written to utilize a discrete, incremental version of this controller during the operation of the absorption/desorption plant.

Figure 8 presents plant data for a typical case in which the set point of the absorber exit concentration is lowered

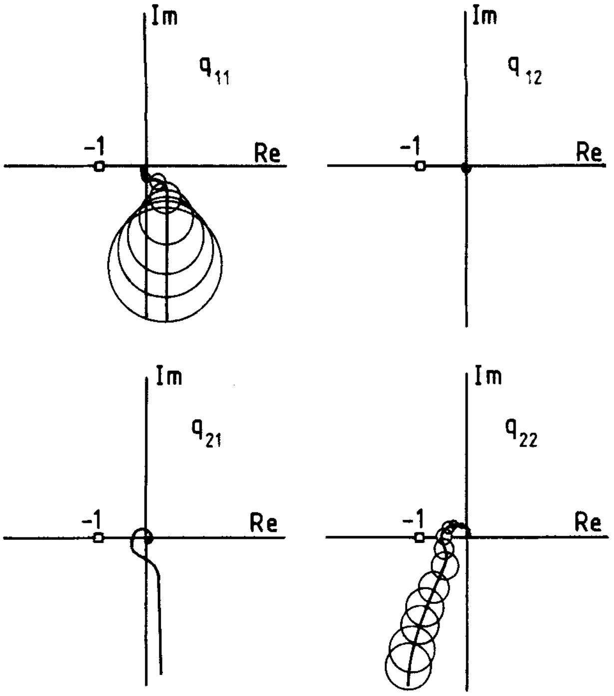

Figure 7. Nyquist array for $K = K_1 K_2 K_3$.

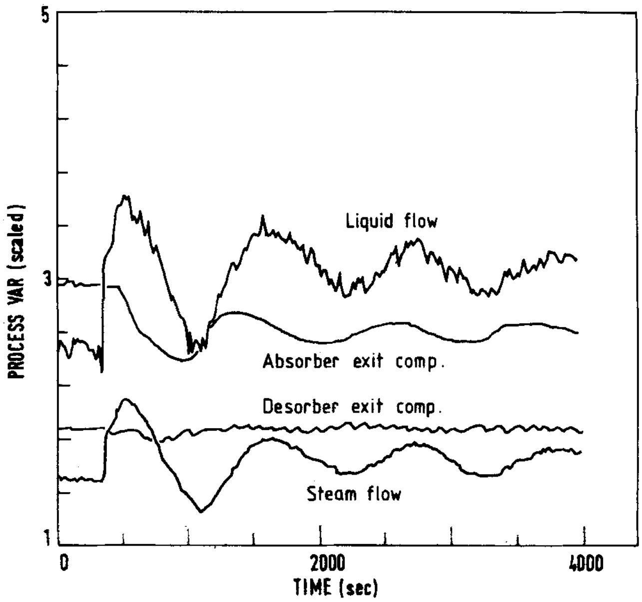

Figure 8. Plant response (multivariable control) to a change in set point.

by 13% in anticipation of a potential future increase in gas load. The set point of the composition of the liquid leaving the reboiler is kept constant. The experimental conditions are similar to those for the model identification experiments.

The results confirm the effectiveness of the system in eliminating the interaction between the two outputs. However, owing to inherent process instabilities associated with the boiling in the reboiler, a high frequency component is superimposed on the output from the desorber. Nevertheless, the controller shows good resilience to disturbances of this kind.

This response (decaying oscillations in the relevant output and both controls) can be compared with a similar experiment using two independent control loops for the

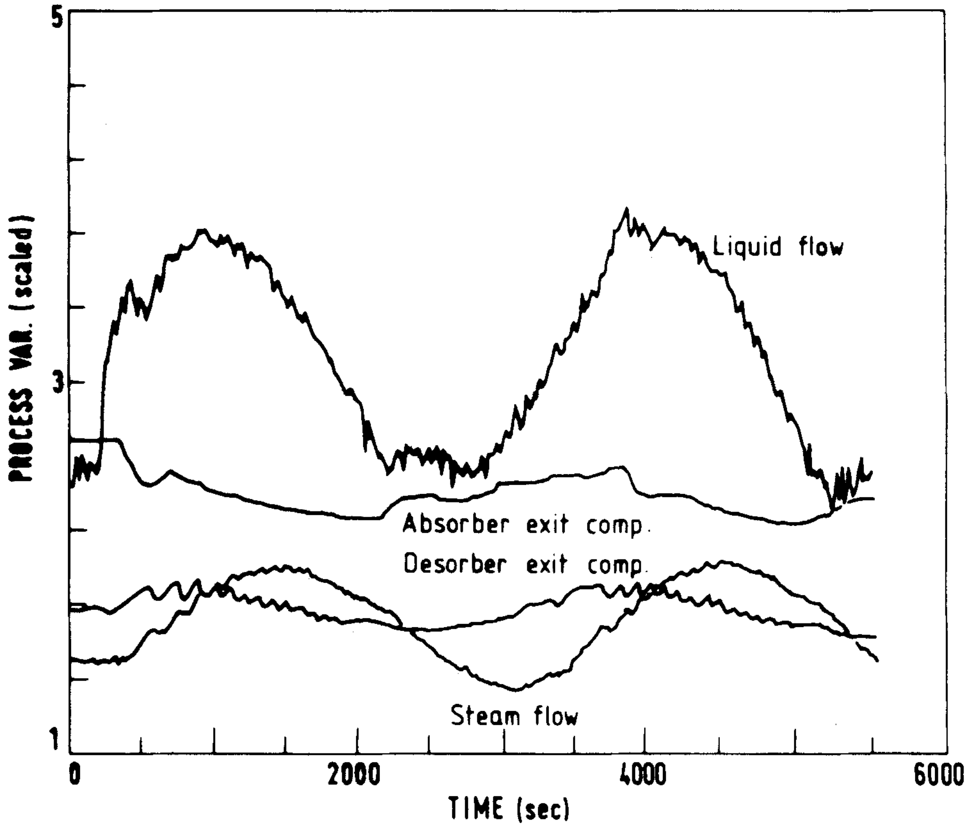

Fig. 9. Plant response (quasi independent single loop control) to a change in set point.

absorber and desorber concentrations. Figure 9 shows the best pair of independent single input-single output, proportional + integral controllers; it is unable to eliminate the nonminimum phase characteristics which arise after the liquid circulation has been increased in response to the change in set point. No amount of on-line tuning of this control algorithm can lead to satisfactory performance.

The nonminimum phase (that is, wrong way) behavior in the open-loop response to a change in liquid circulation rate is an inherent part of the plant's performance and is easily observed in step-response experiments (Albrecht, 1977). With other inputs and controls kept constant, an increase in liquid flow rate will lead to an initial decrease in the concentration of carbon dioxide in the liquid leaving the absorber; however, after a long period of time, because of the additional thermal load on the system, the stripper will be unable to remove the carbon dioxide as effectively, and the carbon dioxide concentration in the liquid will approach a higher steady state value.

Although the multivariable controller outlined in Figure 8 is far from perfect, it does lead to stable, satisfactory control, can be tuned on-line and was realized after a minimum of computational and modeling effort. It is important to point out that the results in Figure 8 represent the implementation of the a priori control strategy predicted by the control system design algorithm. Slight additional improvements in performance can be achieved by ad hoc tuning while the plant is running. Such tuning generally was represented by small changes in the constants predicted by the linear analysis.

Work has continued on two related problems associated with these multivariable control systems: the performance of the controllers when faced with load changes as well as set point changes and the design of controllers when the linearization approximations become less valid (Tranmer, 1980). These results will be reported at a later date.

NOTATION

A	=	polynomial in z^{-1}
a_i, b_i	=	model parameters
B	=	polynomial in z^{-1}
d	=	time delay for system
d_i	=	process time delays
D	=	polynomial in z^{-1}
e_i	=	error sequence (white noise)
F	=	polynomial in z^{-1}, $F = 1/D$
G	=	transfer function matrix
g	=	elements of G
H	=	closed loop transfer function matrix
K	=	controller matrix
L	=	number of delay intervals
N	=	number of data sets
Q	=	system matrix = $G\,K$
T_s	=	sample interval, s
u	=	model input (control)
$u(k)$	=	input at time k
w	=	continous transform variable
y	=	model output
$y(k)$	=	output at time k
z	=	z transform variable
γ	=	polynomial in z^{-1}
Φ	=	autocorrelation of error sequence
ξ_i	=	error sequence
$\xi(k)$	=	model error at time k

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